

TriboSolver

User Manual

Version 3.1

Contents

1. ICONS USED IN THE MANUAL.....	4
2. INTRODUCTION.....	4
2.1 Description.....	4
2.2 Assumptions in use of BEM.....	7
2.3 Validity of BEM for Real Contact Area Simulations.....	8
2.4 Accuracy of the TriboSolver Algorithms.....	8
2.5 Support.....	8
3. GENERAL INFORMATION ABOUT TRIBOSOLVER.....	9
3.1 Schematic Diagram of the Software.....	9
3.2 Load Sharing Concept for Mixed Lubrication Modelling (Load Sharing Loop).....	10
3.3 coordinate system and units.....	11
4. SOLVERS.....	12
4.1 Contact Solver.....	12
4.2 Adhesive Contact Models.....	16
4.3 Electroadhesion Model.....	16
4.4 Lubrication solver.....	17
4.5 Thermal Solver.....	18
5. VERIFICATION USING ANALYTICAL SOLUTIONS.....	19
5.1 Hertz Solution.....	19
5.2 Westergaard Solution.....	20
5.3 Validation for a contact of coated materials.....	20
5.4 Subsurface stress validation for friction free and frictional interface.....	21
5.5 Adhesion and Electroadhesion Verification.....	21

6.	SIMULATION STEPS (DESKTOP VERSION).....	22
6.1	Simulation Type.....	23
6.2	Macro Geometry.....	24
6.3	Micro Geometry.....	25
6.4	Solid Material Model Tab.....	26
6.5	Contact Conditions Tab.....	27
6.6	Adhesion and Electroadhesion Inputs.....	27
6.7	Contact Grid Tab.....	27
6.8	Lubricant Model Tab.....	28
7.	OUTPUT VARIABLES AND RESULT FILES.....	29
7.1	TriboSolver 3.1 result sets.....	29
7.2	Inspecting and saving results.....	30
7.3	Surface preparation and recovered inputs.....	31
7.4	Comparing saved results.....	32
8.	TRIBOSOLVER IN LITERATURE.....	33
9.	APPENDICES.....	33
9.1	Appendix 1.....	33
10.	REFERENCES.....	34

1. **ICONS USED IN THE MANUAL**

Throughout this document, the pictograms below are used to underline points or important notions

	Important information
	Good to know - Tricks
	Risk in front of a parameter setting or of a specific action

2. **INTRODUCTION**

2.1 DESCRIPTION

TriboSolver 3.1 is a desktop application for the simulation of a general tribological contact - a contact of two rough surfaces in relative motion, as schematically depicted in Figure 2-1. It provides six simulation types: Dry Contact, Adhesion, Contact Temperature, Mixed Lubrication, Seizure and Wear. The Adhesion tab independently enables surface adhesion, electroadhesion, or both. The application runs locally on your computer. This manual explains the available models, inputs, outputs and verification basis without assuming familiarity with an earlier release.

Examples of the problems that can be solved with TriboSolver: nominally smooth (elasto) hydrodynamic simulation, nominally smooth and rough contact simulations (pressure and subsurface stress calculations, elastic/plastic deflections, contact area, stiffnesses), Stribeck Curve simulation (calculation of friction in mixed lubricated contacts), calculation of contact temperatures, wear and wear particles simulation and surface roughness evolution, tribofilm evolution.

Solvers are based on the half-space approximation theory for the contact, thermal and hydrodynamic simulations, which makes it faster compared to Finite Element Method or other fully numerical approaches.

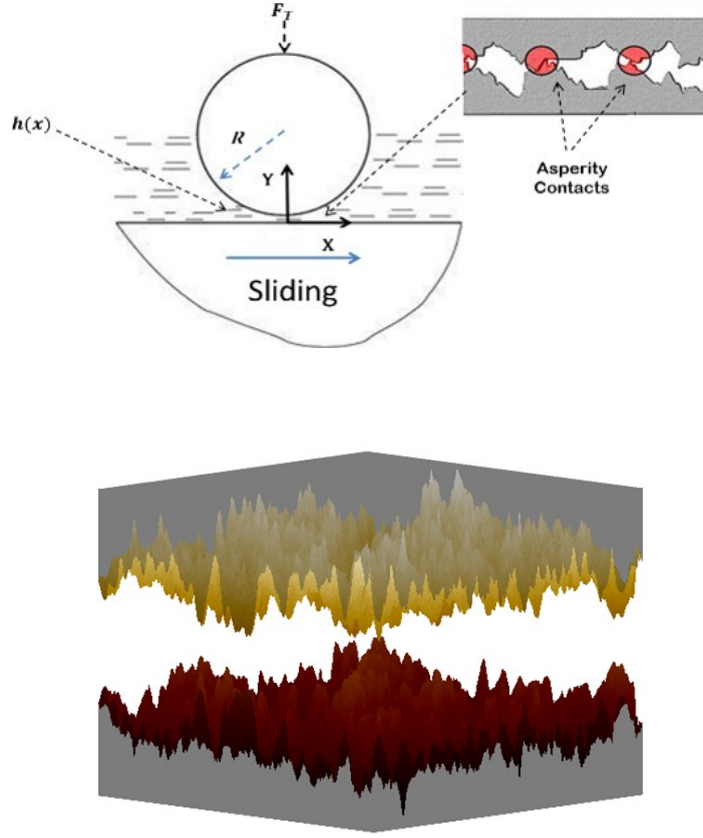


Figure Introduction-1. Tribological contacts.

In general, solution of contact problem requires solution of elasticity theory equations (Timoshenko & Goodier, 1970). Unfortunately, in most cases, solution cannot be obtained analytically and therefore numerical methods have to be used. A straightforward approach would be the use of Finite Element Method (FEM), however, in case of rough surfaces this requires significant computational resources, e.g., time and memory. Thus, FEM is not widely used in the rough surfaces contact analysis.

A widely used approach is on the other hand – Boundary Element Method (BEM). The approach assumes that the contacting bodies are semi-infinite, as shown in Figure Introduction-2. Under the given pressure, an analytical solution of elasticity theory equations linking elastic deflection and contact pressure then can be obtained (Love, 1929):

$$u(x, y) = \int_{\Omega} k(x - \xi, y - \eta) p(\xi, \eta) d\xi d\eta, \quad (1)$$

Where $k(x, y) = \frac{1}{\pi E^*} \frac{1}{\sqrt{x^2 + y^2}}$, and $\frac{1}{E^*} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}$. $u(x, y)$ is the elastic deflection of the surface, E_1, E_2, ν_1, ν_2 are the Yong's moduli and Poisson's ratio of body 1 and 2 correspondingly.

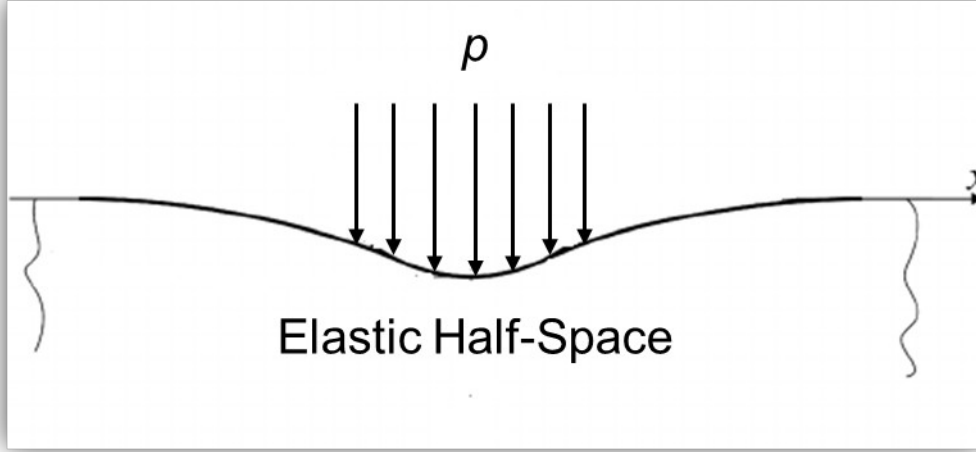


Figure Introduction-2. Semi-infinite elastic half-space

From (1) it can be noticed that the elastic deflection of the surface $u(x, y)$ only depends on the pressure acting on the surface. If the analytical form of the pressure profile is known, then it can be integrated and the complete solution can be obtained, as for example in case of a well-known Hertz solution (Johnson, 1984). In case when the shape of the pressure function is not known, the integral needs to be evaluated numerically. Numerical evaluation of the integral can be done efficiently using Fast Fourier Transformation, since equation (1) is a convolution and thus:

$$u(x, y) = (k * p)(x, y) = F^{-1}[F(k) \cdot F(p)], \quad (2)$$

Where '*' denotes convolution, F - Fourier transform operator and ' $\cdot$ ' is the multiplication operator. The time-consuming convolution operation in real space is substituted with a simple multiplication in Fourier space, which makes the calculation of the integral computationally efficient (the number of operations is decreased from $O(N^2)$ to $O(N \log N)$).

Having the relation between the elastic displacement and pressure given, it is necessary to specify boundary conditions and introduce the surface roughness. For a simple ideally elastic non-adhesive contact with specified load F_C , the following system of equations needs to be solved:

$$\left\{ \begin{array}{l} u(x, y) = \int_{\Omega} k(x - \xi, y - \eta) p(\xi, \eta) d\xi d\eta \\ u(x, y) = z(x, y) - \delta, \forall x, y \in A_c \\ p(x, y) > 0, \forall x, y \in A_c \\ F_C = \iint p(x, y) dx dy \end{array} \right. \quad (3)$$

where $z(x, y)$ is the surface topography, δ is the rigid body displacement, A_c is the contact area. The system is solved for contact area A_c and contact pressure $p(x, y)$. This system of equations is solved using Conjugate Gradient Method (Shewchuk) as initially proposed by (Polonsky & Keer, 1999). This method or its variations are widely used to solve the system of equations for rough surface contacts. The system of equations can be adapted in case if plasticity, coatings, etc. are introduced by altering the kernel $k(x, y)$ or boundary conditions. For example, if the control variable is rigid body displacement δ , the last equation of the system (3) can be omitted.

The use of BEM brings the following advantages as compared to other numerical methods, such as FEM:

- Need to discretize only boundary (fast simulations, low memory requirements)
- Solution of contact problem for rough surfaces is very robust

The fast simulations and robust algorithms made the use of BEM widely spread in contact mechanics and tribology community.

2.2 ASSUMPTIONS IN USE OF BEM

It should be mentioned that the use of BEM is associated with the following assumptions:

- Use of small-strain elasticity approximation
- BEM is not applicable in some cases (e.g., non-homogeneous materials, excluding coatings)
- Use of semi-infinite half-space approximation
- Deformations are relatively small

Small strain approximation comes from the use of the equations of the classical theory of elasticity. The theory is used to obtain the analytical solution (1). This assumption is also used in other numerical methods, such as FEM, thus is not BEM specific.

The use of BEM is restricted to the cases where the fundamental analytical solutions in integral form, as given by (1), can be obtained. This is not always possible, as in case of non-homogeneous materials. It should be noted however, that in the recent years, the BEM approach was extended to consider multi-layered coatings with uniform mechanical properties, gradient materials and even for materials with inclusions, therefore covering many areas of practical interest.

And last, the fundamental analytical solutions necessary for BEM to be utilized assume semi-infinite bodies, and therefore neglect the impact of boundaries on the deformation of the surfaces. This may lead to additional inaccuracies in the calculations.

2.3

VALIDITY OF BEM FOR REAL CONTACT AREA SIMULATIONS

The results of BEM have been documented to match well with the analytical solutions and in some cases, experimental data. Experimental verification of the solution of the contact mechanics equations is typically associated with the measurement of the contact area, which is also frequently the quantity of interest (Jacobs & Martini, 2017). However, existing measurement techniques do not allow to measure the contact area down to the needed resolution for many practical cases, such as a contact of rough elastic surfaces. At the same time, the correlation between the contact area calculated using BEM and friction coefficient is widely reported (Jacobs & Martini, 2017)(Lechthaler, Ochs, Mücklich, & Dienwiebel, 2019).

For soft materials, direct measurements of real contact area are possible. Recent contact mechanics challenge results revealed that the simulations of contact area and the measured values can match well for the soft materials (Müser et al., 2017). The work performed by Bart Weber at ARCNL also indicates a good qualitative agreement of TriboSolver contact area calculation in a contact of glass and plastic sphere (Weber, Suhina, Brouwer, & Bonn, 2019).

2.4

ACCURACY OF THE TRIBOSOLVER ALGORITHMS

The algorithms used in TriboSolver are verified using various analytical models and experiments available in the literature, where possible (see sections below). This includes Hertz, (for contact pressure, as well as subsurface stresses), Westergaard (Rey, Anciaux, & Molinari, 2017) and other well-known analytical solutions. This comparison will be shown in the sections below.

2.5

SUPPORT

If there are any questions or suggestions, please contact administration@tribology.eu.

3.

GENERAL INFORMATION ABOUT TRIBOSOLVER

3.1

SCHEMATIC DIAGRAM OF THE SOFTWARE

The major function of the simulator is to calculate contact conditions in a tribological system. The schematic simulation algorithm is shown in Figure 3-1. The Contact and Hydrodynamic Solvers form an iterative load-sharing stage when mixed lubrication is selected. The resulting contact pressure and lubricant-film state are passed to the Thermal Solver, then to the Seizure Solver and finally to the Wear Solver. Wear updates the surfaces and the cycle is repeated for the next step. Dry Contact and other shorter workflows stop at the appropriate earlier stage. Adhesion is a top-level user choice with independently enabled surface-adhesion and electroadhesion laws.

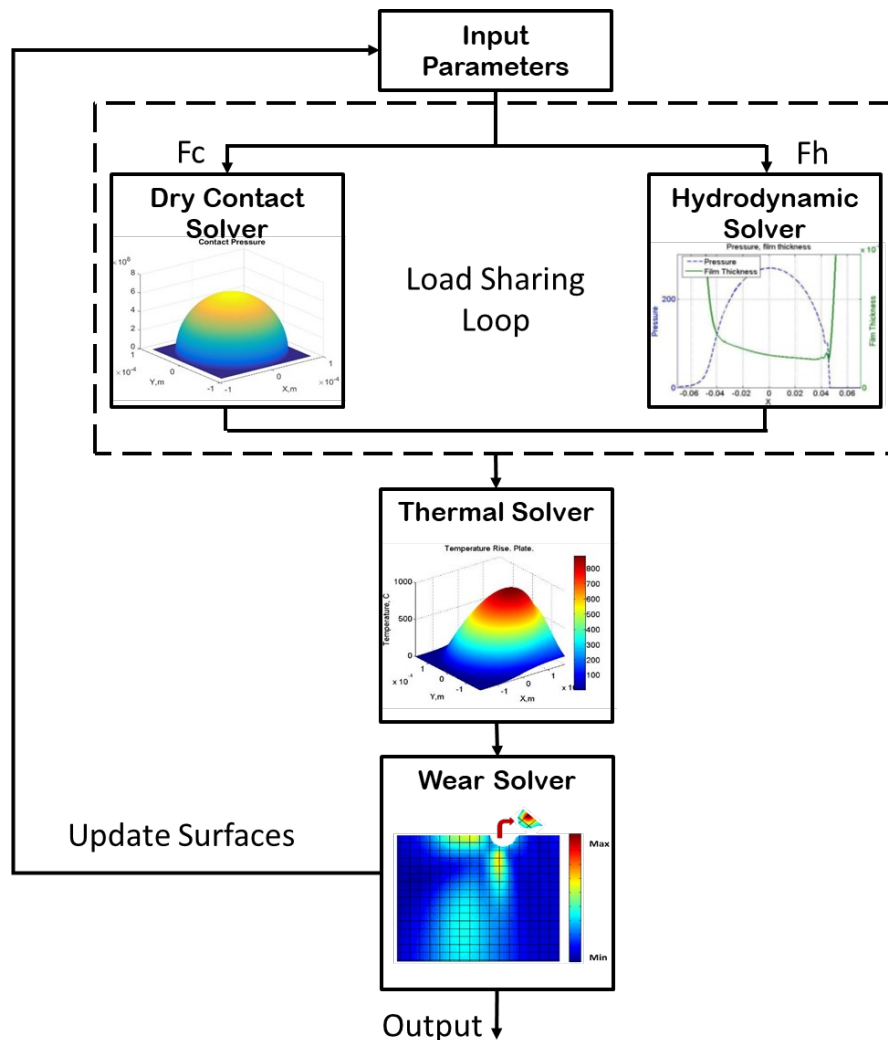


Figure General Information About TriboSolver-3. The layout of TriboSolver

3.2

LOAD SHARING CONCEPT FOR MIXED LUBRICATION MODELLING

(LOAD SHARING LOOP)

The load sharing route in the friction prediction in mixed lubricated contact was established by (Johnson, Greenwood, & Poon, 1972). This theory suggests that the total load is proportionally shared between the lubricant and the surface contacts. This approach is a combination of independent models of smooth Elastohydrodynamic Lubrication (EHL) and dry rough surface contact, which are mutually linked through the proportionality of the carried load. The EHL equations are solved under assumption of smooth surfaces and the surface roughness response is calculated with a separate dry contact model. This makes the problem much easier to handle, as the smooth EHL equations can be solved accurately without convergence problems. These models are widely used for assessment of friction.

The load sharing concept is needed to identify the part of load carried by dry contact (or hydrodynamic, since the total load equals to the sum of dry and hydrodynamic loads).

According to this concept, total load F_T is partly carried by the asperities F_C and partly by the hydrodynamic film F_H :

$$F_T = F_C + F_H \quad (4)$$

If F_C is assumed (which means F_H is calculated), then film thickness can be calculated using hydrodynamic solver. Once the film thickness $h(x, y)$ is known, separation h_s is calculated:

$$\iint_A h(x, y) dA = \iint_{\tilde{A}} (h_s(x, y) - z(x, y) + u(x, y)) d\tilde{A} \quad (5)$$

where $\tilde{A}$ is the lubricated area. Equation (5) follows the definition of the separating distance of Johnson. It should be mentioned, that the total displacement of the surfaces is a sum of the deflection $u(x, y)$ due to contact pressure and the deflection $u_H(x, y)$ due to hydrodynamic pressure, $z(x, y)$ is the surface roughness profile, see Figure General Information About TriboSolver-4.

For a given separating distance h_s between the surfaces, the deflection is:

$$u(x, y) = \delta(x, y), \quad \forall x, y \in A_c \quad (6)$$

where A_c is the contact area, $\delta(x, y) = z(x, y) - h_s(x, y)$. The contact area A_c is actually not known a priori. The boundary condition reads $p(x, y) \geq 0$. A schematic diagram of the contact is shown in Figure General Information About TriboSolver-4.

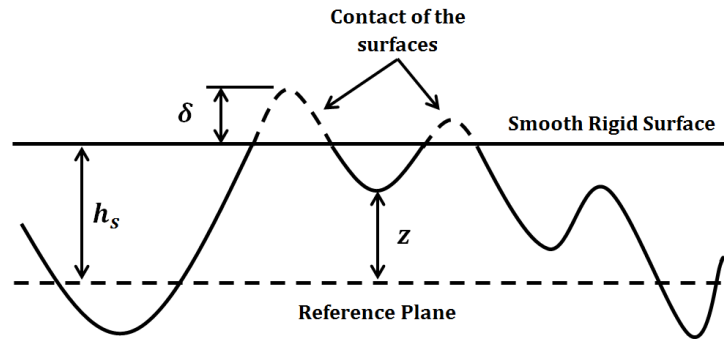


Figure General Information About TriboSolver-4. Schematic diagram of the surfaces in contact.

Based on the separation distance, the load carried by the surface contacts $\tilde{F}_c$ can be calculated using one of the available contact models, as discussed below. If the calculated $\tilde{F}_c$ matches with the assumed F_c , then the equilibrium load sharing is found. If not, the procedure starts over again, until the match is found.

In this section, the generalized algorithm is given. On the other hand, under certain conditions simplifying assumptions can be applied and the models can be simplified. These simplifications are discussed in the sections below.

3.3 COORDINATE SYSTEM AND UNITS

The coordinate system is chosen as shown in Figure General Information About TriboSolver-5. The x,y coordinates for both bodies are the same, while the z coordinates are directed in opposite direction. Besides, there is a convention to name the upper body as Body 1, and the lower body as Body 2. The associated properties use the subscripts or explicitly state, which body is considered. SI units are used for all the properties.

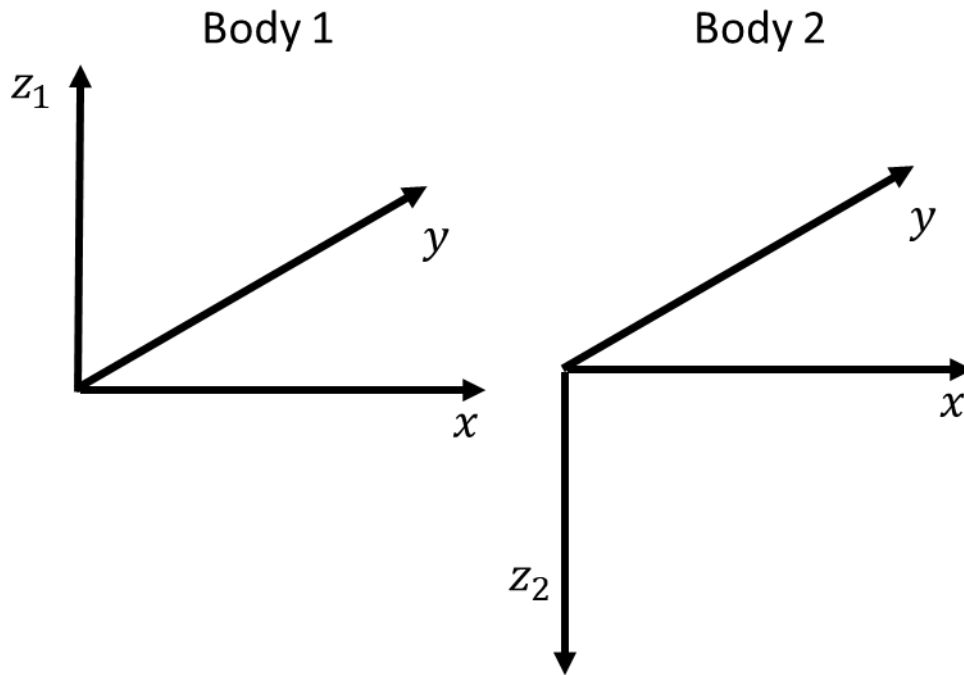


Figure General Information About TriboSolver-5. Coordinate system for bodies in contact

4. SOLVERS

4.1 CONTACT SOLVER

The Contact Solver calculates pressure, deformation, gap, contact area and related contact quantities. TriboSolver 3.1 retains the deterministic asperity-based and half-space BEM models described below and extends the same unified solver boundary with homogeneous and layered materials, measured or generated roughness, subsurface stress and pre-sliding. The optimized non-adhesive Contact Solver is kept separate from the nonlinear Adhesion Solver so ordinary contact is not slowed by inactive attractive laws. Both publish the same pressure, geometry, deformation, gap, load and convergence contract for downstream calculations. The numerical model is selected automatically by the backend from the chosen simulation type. Downstream Thermal, Seizure and Wear calculations consume the pressure and gap fields returned by this common Contact Solver rather than maintaining separate contact implementations.

Asperity is defined as a point on a surface, which is higher than its 9 closest neighbours (other definitions of asperities also exist). Asperity based models substitute the real measured surface topography by a set of asperities of simplified geometry (spherical or elliptical) and assume that they deform independently. These assumptions allow calculation of load carried by surface contacts through analytical Hertz theory. Further description can be found using following link:

<http://www.tribonet.org/wiki/greenwoods-contact-model/>.

An alternative to the description of the surface by means of asperities is the direct use of the (measured) height profile. The half-space approximation can be used to calculate the surface deflection corresponding to the applied pressure, as discussed below.

	Properties of asperities such as radius of curvature, height, asperity density vary with the change of the surface roughness measurement resolution due to the definition of the asperity. The higher is the resolution, the smaller is the radius of curvature and larger is the asperities density. This is a fundamental limitation of the asperity-based models. These models can be used in comparative and qualitative studies. If quantitative information is needed, the half-space-based approach has to be used.
---	---

4.1.1 *Deterministic asperity-based model*

The deterministic model regards the surface as a discrete set of hemi-spherical asperities. Each of them is characterized by radius of curvature β_i and asperity height z_i . By assuming independent asperity deformations, it is possible to calculate the load carried by each asperity F_{ci} using Hertz theory:

$$F_{ci} = \frac{2}{3} E' \beta_i^{0.5} \delta_i^{1.5} \quad (7)$$

where $2/E' = (1 - v_1^2)/E_1 + (1 - v_2^2)/E_2$, with E_1, E_2 – elastic moduli of the surfaces, v_1, v_2 Poissons ratio, $\delta_i = z_i - h_s$. The total load carried by asperities is then $\tilde{F}_c = \sum_i F_{ci}$.

In this form, the model was introduced by Irinel Faraon. This model was further improved by introduction of elasto-plastic deformations of a single asperity by Zhao for statistical asperity-based approach. However, it's implementation for the case of deterministic approach is straightforward. Depending on the value δ_i , asperity deformation is considered to be in fully elastic, elastic-plastic or fully plastic regimes. In all these regimes, equations for the calculation of the contact area and carried load are different. Exact equations can be found in Appendix 1.

In case if there is no transition to the plastic regime, then the elastic-plastic model reduces to the fully elastic case.

	Deterministic models can be used, if the distribution of surface heights does not follow Gaussian distribution or elastic-plastic contact has to be considered.
---	--

4.1.2 *Half-space-based contact model (BEM)*

The corresponding system of equations to be solved to find the load carried by the contact of the surfaces reads:

$$\begin{cases} u(x, y) = \frac{2}{\pi E'} \iint \frac{p(x', y')}{\sqrt{(x' - x)^2 + (y' - y)^2}} dx' dy' \\ u(x, y) = z(x, y) - h_s(x, y), \quad \forall x, y \in A_c \\ p(x, y) > 0, \quad \forall x, y \in A_c \\ p(x, y) \leq P_u \end{cases} \quad (8)$$

where P_u is the lowest hardness of the contacting materials, beyond which the yielding is initiated.

Once the pressure $p(x, y)$ is known, the load carried by surface contacts $\tilde{F}_c$ can be obtained:

$$\tilde{F}_c = \iint p(x, y) dx dy \quad (9)$$

In the system (8), $p(x, y)$ and A_c are unknowns, whereas $h_s(x, y)$ is related with the hydrodynamic film thickness, as discussed above. The pressure $p(x, y)$ outside of the contact area is zero. The set of equations cannot be solved analytically in general and numerical methods have to be used. It also should be noted, that the last equation in the system is used to model perfectly plastic behavior of the material. In the case of pure elastic simulations, this condition should be eliminated from the system of the equations.

Computational time necessary for the solution of the system (8) is dominated by the calculation of the integral equation. This is a convolution integral, and thus, the computational burden can be decreased from $O(N^2)$ to the $O(N \log N)$ by discrete fast Fourier transformation technique (DC-FFT). In this case, the integral equation is rewritten in the following form:

$$u(x, y) = K \otimes p \quad (10)$$

where K is the influence matrix.

The foundation of the numerical solver of the system (8) is the method developed by Polonsky and Keer (Polonsky & Keer, 1999) and is based on the conjugate gradient method. The separating distance must fulfil the condition that the total lubricant volume between the smooth surfaces is equal to that between rough surfaces, as it was defined by Johnson (Johnson et al., 1972). Therefore, the method of Polonsky and Keer was adapted to solve the considered system. The details of the algorithm are given in reference.

Calculation of the subsurface stresses is performed using the theoretical background introduced by (Jacq, Nélías, Lormand, & Girodin, 2002). In this case, the subsurface stress can be calculated using following relation:

$$\sigma_{ij}(x, y, z) = \iint p(\xi, \eta) F_{Nij}(x - \xi, y - \eta, z) d\xi d\eta + s(\xi, \eta) F_{Sij}(x - \xi, y - \eta, z) d\xi d\eta \quad (11)$$

where F_{Nij} , F_{Sij} – are the influence functions corresponding to normal and tangential directions, $p(\xi, \eta)$, $s(\xi, \eta)$ are the normal and tangential stresses on the surface. Indices $(i, j) \in \{x, y, z\}$ are

used to show the stress components. These integrals are also convolutions and can be calculated using the DC-FFT based approach:

$$\sigma_{ij}(x, y, z) = F_N \otimes p + F_S \otimes s \quad (12)$$

The details of the derivation and the functions can be found in the references. In case of layered surfaces, these functions are given in frequency domain.

4.1.2.1 Plasticity Model

With respect to plasticity simulation, it is assumed that the material behaves as an elastic-ideally plastic material, as shown in Figure Solvers-6. In practice this means that the system of equations (3) gets an additional boundary condition:

$$p(x, y) \leq P_u \quad (13)$$

where P_u is the lowest hardness of the two contacting materials.

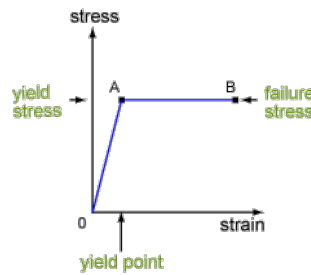


Figure Solvers-6. Elastic-ideally plastic material model

4.1.2.2 Friction Model

The simplest relation between the friction force F_f and real contact area is given as follows:

$$F_f = \tau_{eff} A_r \quad (14)$$

Where τ_{eff} is the effective shear strength and A_r is the real contact area. The A_r can be calculated using TriboSolver and later on multiplied by the effective shear strength to obtain friction force. Note however, that the presence of the shear force may impact the contact area, but especially the subsurface stress distribution, see the validation section, Figure Verification using analytical solutions-10. As it can be seen, the subsurface stress maximum values increase, but what is even more important, the location of the maximum subsurface stresses moves up, towards the surface. This will have a large impact on the mechanical wear/transfer processes.

In TriboSolver, the Coulomb friction model is used. The model simply introduces a shear stress on the surface as a function of pressure as follows:

$$\tau = p \cdot \mu \quad (15)$$

Where μ is the user specified friction coefficient. This model assumes local variation of the shear stress on the surface as a function of local pressure.

4.2

ADHESIVE CONTACT MODELS

Adhesive contact allows tensile surface traction outside the hard-wall contact region. The mechanical equilibrium therefore contains both a non-negative repulsive pressure and a non-positive adhesive pressure. TriboSolver reports the total pressure as their pointwise sum and separately integrates the repulsive, adhesive and total normal loads. Adhesive models remain within the Contact Solver, so the same material, macrogeometry, roughness, numerical grid and result contracts are used by the other simulation stages.

$$p_{total}(x, y) = p_{repulsive}(x, y) + p_{adhesive}(x, y)$$

$$F_{total} = F_{repulsive} + F_{adhesive}$$

4.2.1

FFT-BEM exponential adhesion for rough surfaces

For rough surfaces, TriboSolver uses the exponential interaction law described by Bugnicourt et al. (2018). The applied normal load is prescribed, and elastic deformation is solved together with repulsive and adhesive pressure. Results include total and component pressures, displacement, physical gap, contact area, load components and convergence information.

For work of adhesion w , physical gap g and interaction range ρ , the interaction energy per unit area U and signed adhesive pressure are:

$$U(g) = -w \exp\left(\frac{-g}{\rho}\right)$$

$$p_{adhesive}(g) = -\frac{w}{\rho} \exp\left(\frac{-g}{\rho}\right)$$

Adhesive traction can vary over molecular or sub-grid length scales. The interaction range must therefore be resolved by the numerical grid, and both grid and domain convergence must be checked. TriboSolver emits a resolution warning when the interaction range is smaller than one grid cell. Integrated load may appear stable while the tensile-pressure peak is not yet grid converged, so both field and scalar outputs should be reviewed.

4.3

ELECTROADHESION MODEL

The Electroadhesion model follows the gap-capacitor formulation used by Peng et al. (2026). It is coupled to the unified Contact Solver: the mechanical model first calculates the hard-wall contact mask and non-negative physical separation in the open gap. Electrostatic attraction is evaluated only in non-contacting gap cells; solid-solid contact cells are excluded from the electrical traction calculation.

For applied interface voltage V , local physical gap g , dielectric-layer thickness t_i and relative permittivity κ_i , the governing quantities are:

$$d_{eff} = \frac{g}{\kappa_g} + \sum_i \frac{t_i}{\kappa_i}$$

$$D = \frac{\varepsilon_0 V}{d_{eff}}$$

$$E_g = \frac{D}{\varepsilon_0 \kappa_g}$$

$$p_e = \frac{D^2}{2 \varepsilon_0 \kappa_g}$$

$$F_e = \int_A p_e dA$$

The force integral is evaluated over electrically active, non-contacting cells. In the ideal-capacitor mode, force is symmetric in voltage polarity and proportional to the square of the applied voltage. The load-controlled Adhesion solver evaluates repulsive contact, surface adhesion and Peng electroadhesion in the same nonlinear conjugate-gradient iteration. At each iteration it updates the physical gap, evaluates the enabled attractive laws, and balances their signed pressure with the prescribed external load. For electroadhesion alone, the load and friction relationships are:

$$F_{repulsive} = F_{external} + |F_e|$$

$$F_{friction} = \mu_0 F_{repulsive}$$

$$\mu_{apparent} = \frac{F_{friction}}{F_{external}}$$

$$C_{apparent} = \left| \frac{Q_{total}}{V} \right|$$

Treatment of the contact discontinuity. The Peng electrical law is defined only in the open gap. TriboSolver clamps the converged physical separation to $g \geq 0$ and uses the repulsive hard-wall active set to identify mechanical contact. For every cell with $g = 0$, the electric field, free surface-charge density and electroadhesive traction are set exactly to zero. For every cell with $g > 0$, they are evaluated from the local series dielectric thickness shown above.

4.4 LUBRICATION SOLVER

For the hydrodynamic simulations, in TriboSolver Moes fit (there are also other fits exist) to calculate the central film thickness. In situations, when the film is parallel in the contact (highly deformed contacts), the central film is a representative value (since the film thickness is almost not changing within the contact) and can be used for calculation of separation. In such contacts, the hydrodynamic pressure is also well approximated by the Hertz pressure profile (and the contact

area as well). The hydrodynamic friction force is only approximated. This approach is significantly faster than the fully numerical approach.

4.4.1 Moes function fit

A Moes fit calculates the central film thickness as a function of the parameters according to the following equation:

$$H_C = \left[\left(H_{RI}^{7/3} + H_{EI}^{7/3} \right)^{3s/7} + \left(H_{RP}^{7/2} + H_{EP}^{7/2} \right)^{-2s/7} \right]^{s^{-1}} \quad (16)$$

Where H_C is the dimensionless central film thickness. Dimensionless central film thickness is represented through dimensionless variables introduced by Dowson and Higginson:

$$U_\Sigma = \frac{2U\mu}{E'R}, W = \frac{F_H}{BE'R}, G = \alpha E' \quad (17)$$

Where B is the line contact width and α is pressure viscosity coefficient. Further, dimensionless variables introduced by Moes are used:

$$H_C = \frac{h_c}{R} U_\Sigma^{-1/2}, M = W U_\Sigma^{-1/2}, L = G U_\Sigma^{1/4} \quad (18)$$

Expressions for the terms in (16) are given by:

$$\begin{aligned} H_{RI} &= 3 M^{-1} \\ H_{RP} &= 1.287 L^{2/3} \\ H_{EI} &= 2.621 M^{-1/5} \\ H_{EP} &= 1.311 M^{-1/8} L^{3/4} \end{aligned} \quad (19)$$

Further details regarding the film thickness calculations using the central film thickness fits can be found on TriboNet for [line](#) and [point](#) contacts. Links contain all the equations as well as the online tools for the calculations.

4.5 THERMAL SOLVER

In case if the heat source q is known, it is possible to calculate the temperature rise in the solid, according to the half-space approximation (using the fundamental solution of heat transfer equations for the semi-infinite solid, (Bos & Moes, 1995)):

$$\theta(x, y, z, t) = \iint_{S_q} \int_0^t q(x', y', \tau) \cdot G(x-x', y-y', z, \tau) d\tau dx' dy' \quad (20)$$

Where G is the fundamental solution of heat transfer equations:

$$G(x-x', y-y', z, \tau) = \frac{1}{4(\pi\kappa\tau)^{3/2}} e^{\left\{ -\frac{(x-x'-U\tau)^2 + (y-y')^2 + z^2}{4\kappa\tau} \right\}} \quad (21)$$

The total heat source is known, if the contact pressure, boundary friction coefficient μ and sliding speed U are known:

$$Q(x, y) = \mu p(x, y) U \quad (22)$$

Part of the total heat goes to one body and the left to another. Partition of the heat is found from the condition of the equality of surface temperatures in contact:

$$\theta_1|_{S_q} = \theta_2|_{S_q} \quad (23)$$

More details on the thermal simulations can be found in (Bos & Moes, 1995).

5. VERIFICATION USING ANALYTICAL SOLUTIONS

5.1 HERTZ SOLUTION

As discussed in earlier the contact pressure/contact displacement and contact area are found by solving equations (3) in TriboSolver. Validation of the contact model can be performed by comparison to analytical solutions, particularly Hertz solution, see Figure Verification using analytical solutions-7. The agreement is good within 0.1%. Note that sufficient resolution is needed to get a good agreement. Hertz solution is obtained for a contact of a sphere with a flat. See further details on Hertz theory [here](#).

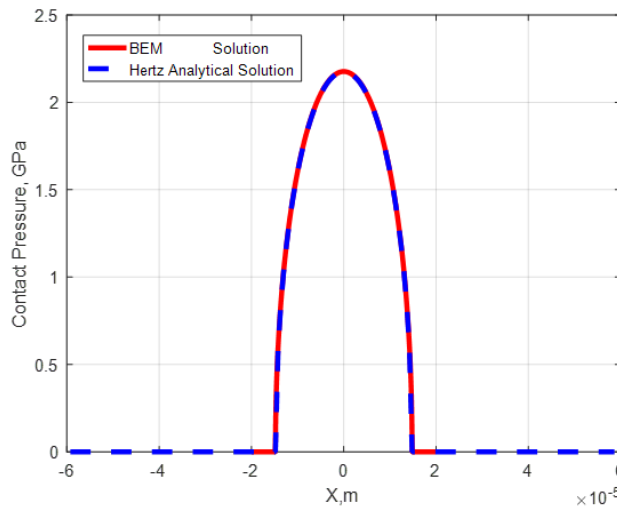


Figure Verification using analytical solutions-7. Pressure profiles, TriboSolver vs Hertz solution

5.2

WESTERGAARD SOLUTION

In case of a contact of a sinusoidal profile with a rigid flat, Westergaard obtained analytical solution for the full contact situation (Yastrebov, Anciaux, & Molinari, 2014). The comparison is shown in Figure Verification using analytical solutions-8. The agreement is good within 0.1%. Note that sufficient resolution is needed to get a good agreement.

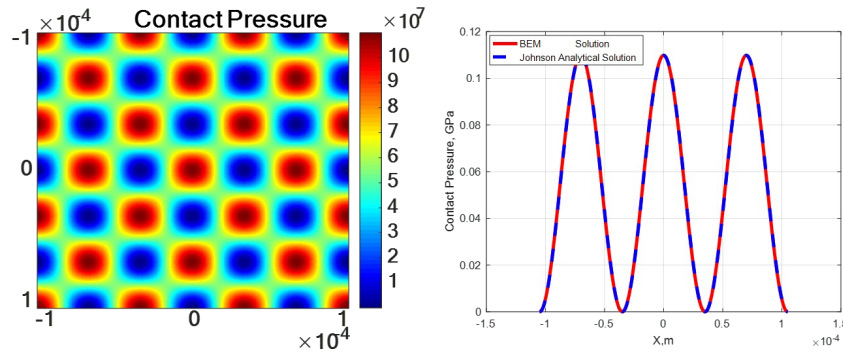


Figure Verification using analytical solutions-8. Pressure profiles, Westergaard solution for full contact

5.3

VALIDATION FOR A CONTACT OF COATED MATERIALS

In case of a contact of coated materials, the validation was performed using the simulation profiles obtained from literature, see Figure Verification using analytical solutions-9.

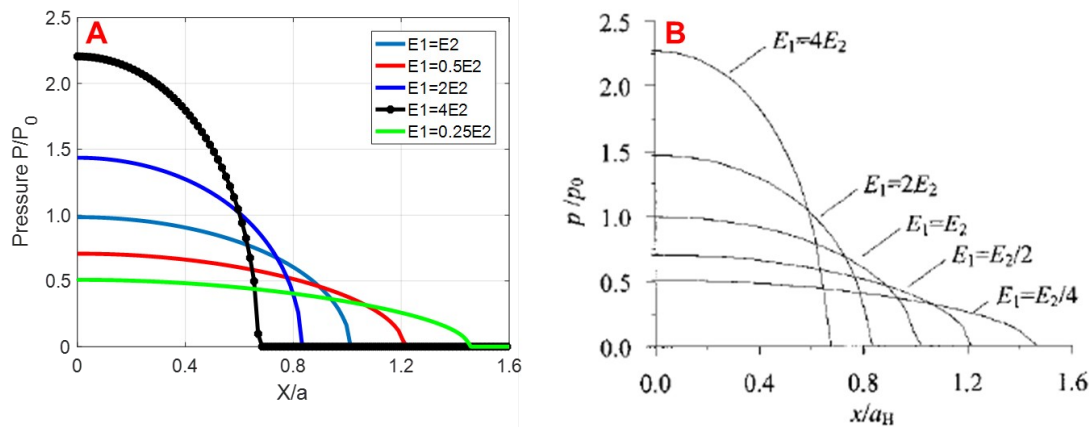


Figure Verification using analytical solutions-9. Simulation of contact pressure for coated materials with various coating properties ratio, A – TriboSolver, B – literature data (Liu & Wang, 2002).

5.4

SUBSURFACE STRESS VALIDATION FOR FRICTION FREE AND FRICTIONAL INTERFACE

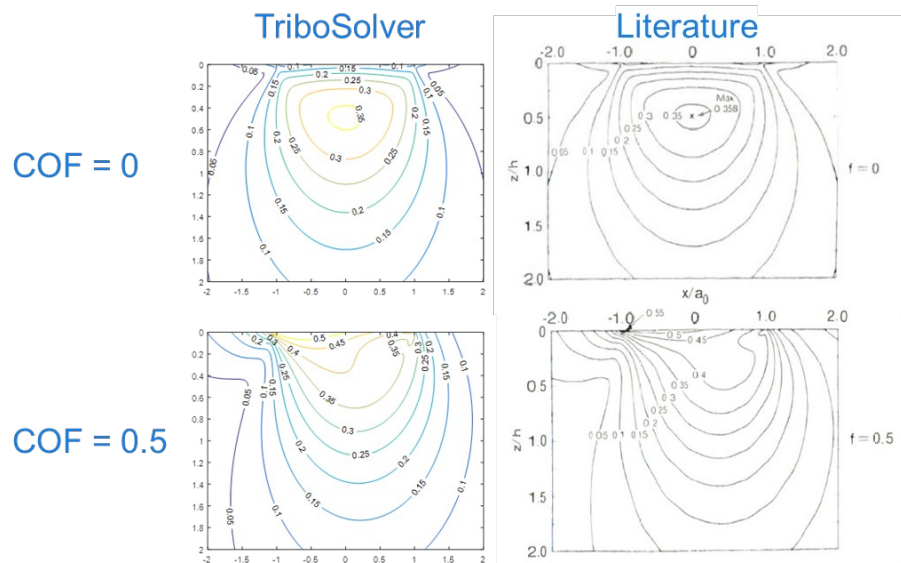


Figure Verification using analytical solutions-10. Von Mises subsurface stress profile cross-section, comparison of TriboSolver and literature (King & Sullivan, 1987).

5.5

ADHESION AND ELECTROADHESION VERIFICATION

Figure 5-5 compares TriboSolver 3.1 with a reference calculation of the load-controlled exponential adhesion formulation of Bugnicourt et al. (2018), equations 27 and 33–38. This case is available through Adhesion with surface adhesion enabled, electroadhesion disabled and no microgeometry. It checks agreement with a reference evaluation of the same equations; it is not experimental validation or a reproduction of a published figure.

Both calculations use a 5 mm sphere on a flat counterbody, normal load 1 mN, 64×64 points over a $400 \times 400 \mu\text{m}$ periodic domain, and identical materials with Young's modulus 45.5 MPa and Poisson's ratio 0.3 (reduced modulus 25 MPa). The work of adhesion is 5 mJ/m^2 and interaction range $0.2 \mu\text{m}$. The solver tolerance is 10^{-5} with a maximum of 3,000 iterations.

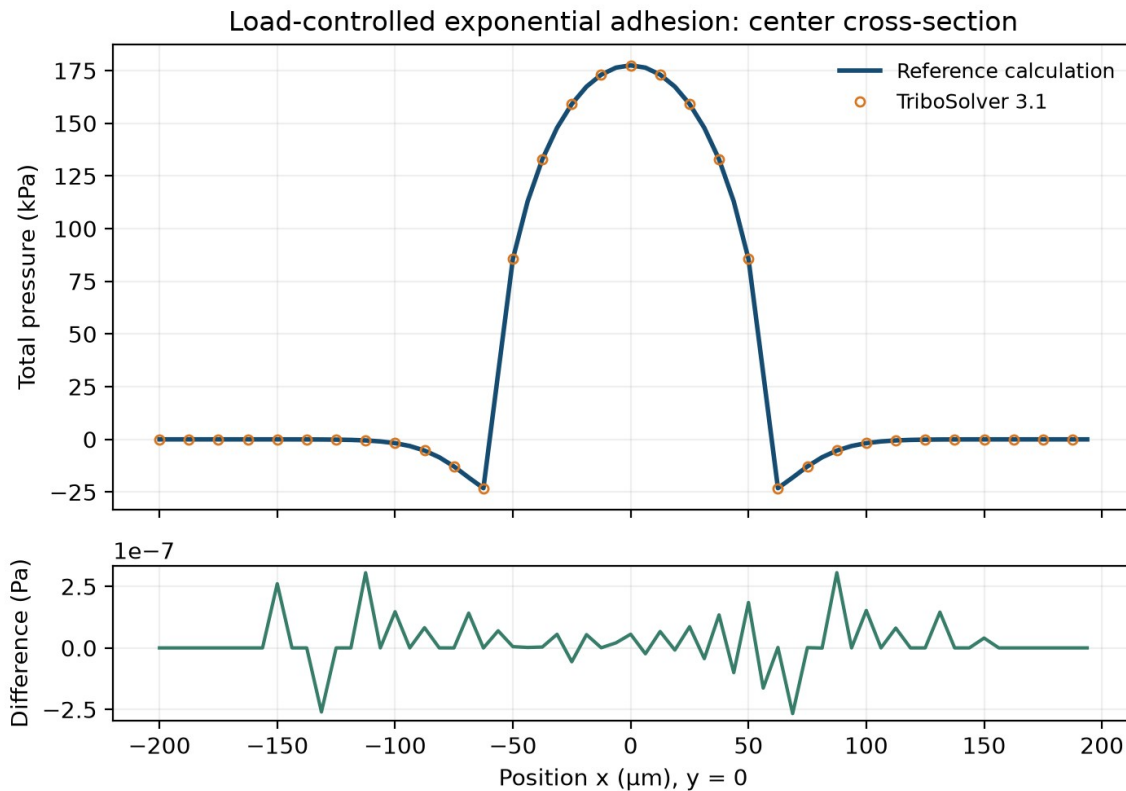


Figure 5-5. Prescribed-load surface adhesion: pressure at $y = 0$ and the application-minus-reference difference. Compression is positive; attraction is negative.

Quantity	TriboSolver 3.1	Reference	Unit
Total normal load	1.000008328	1.000008328	mN
Repulsive load	1.493232650	1.493232650	mN
Surface-adhesion load	-0.493224321	-0.493224321	mN

Both calculations converge in 194 iterations. Across all 4,096 cells, the maximum absolute total-pressure difference is 4.87×10^{-7} Pa; the maximum gap difference is 3.30×10^{-20} m. The recovered normal load differs from the prescribed load by about 0.00083%. The table rounds the values for readability.

Electroadhesion is checked separately against the parallel-plate capacitor equations, voltage-squared scaling, polarity symmetry, dielectric-layer equivalence and exclusion of electrical traction from zero-gap contact. These checks do not establish a rerun of the rough-contact experiments of Peng et al. (2026), whose measured surface maps are unavailable. Figure 5-5 and the table cover surface adhesion only.

6. SIMULATION STEPS (DESKTOP VERSION)

This chapter describes the TriboSolver 3.1 setup sequence. Navigation pages and input rows appear according to the selected analysis and options. Screenshots show the current desktop interface; the visible controls depend on the chosen settings.

Enter the physical quantities through the desktop interface and save the complete setup in a versioned .tribo.json project. Controls that do not apply to the chosen simulation route are hidden so that every visible field contributes to the active calculation.

Every displayed value must be checked together with its unit. The active numerical grid is entered directly in X and Y. Maximum grid points per axis and the remaining solver tolerances are configured under Settings > Numerical settings. The default maximum is 2048 points per axis; this is a configurable ceiling, not a recommended grid for every calculation. Memory and calculation time remain practical limits.

If Run rejects a load, elasticity modulus, Poisson ratio or domain length, the message identifies the page, field and required value. Close the message to edit the selected field, then Run again. Cancel displays a cancellation request; an active solver operation may take several seconds to stop. The cancellation message remains visible until completion. A cancelled run retains the previous results and does not save a new result. On Simulation Type, scroll to Review active setup to check the current inputs for the next run, including body properties, domain, grid and available subsurface/coating selections. Existing results retain their original inputs.

6.1 SIMULATION TYPE

The very first step in setting up the simulation in TriboSolver is to select the simulation type. There are different possibilities, e.g., Dry Contact Simulation, Thermal Simulation or Mixed Lubrication Simulation. Depending on the choice made on this tab, there will be different options available on other tabs.

Dry contact simulation assumes either a problem without any lubricant at all or a case of boundary lubrication, where the effect of lubricant is limited to a reduction in boundary friction coefficient, while the hydrodynamic effects can be neglected.

Mixed lubrication simulation assumes that surface asperities are not negligible as compared to the lubricant film, however, it is assumed that asperities do not change the shape of the lubricant film. In this case, the load sharing concept refer to earlier can be used.

On this tab the subsurface simulation checkbox can be chosen.

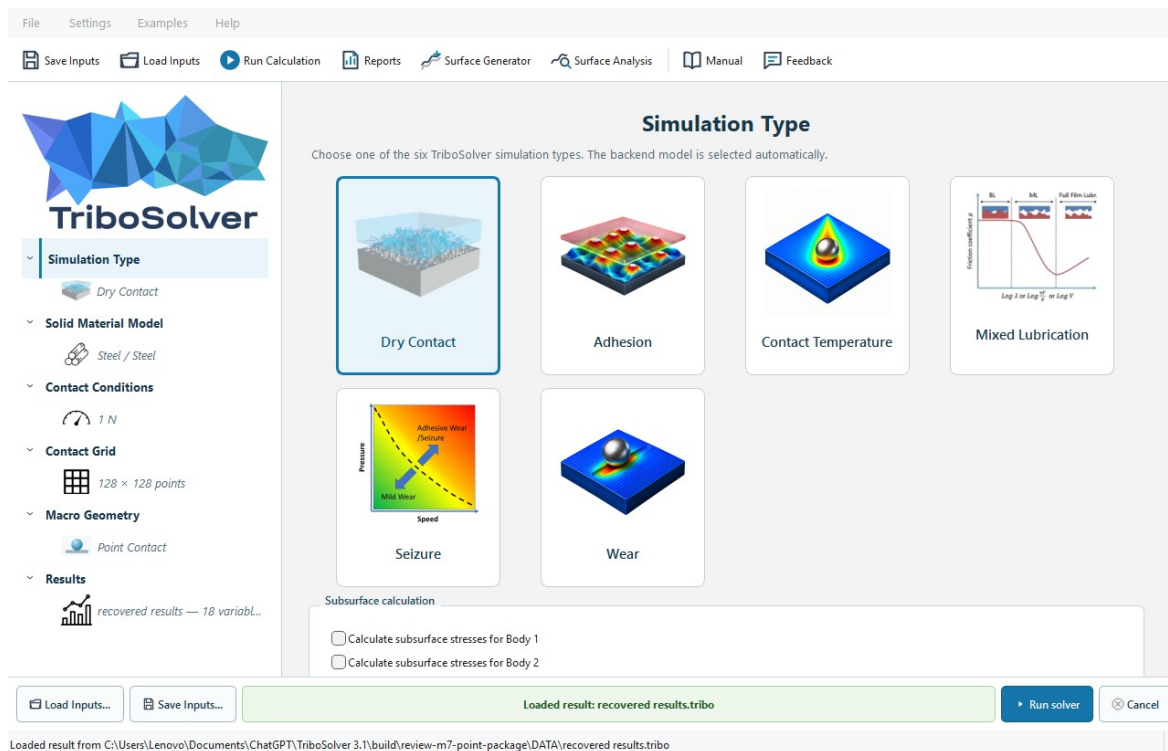


Figure 6-1. Simulation Type: select the analysis and optional subsurface calculation.

6.2 MACRO GEOMETRY

On the Macro Geometry Tab, the type and the dimensions of the Macroscopic geometry is set. Depending on the choice of the geometry, the input fields vary.

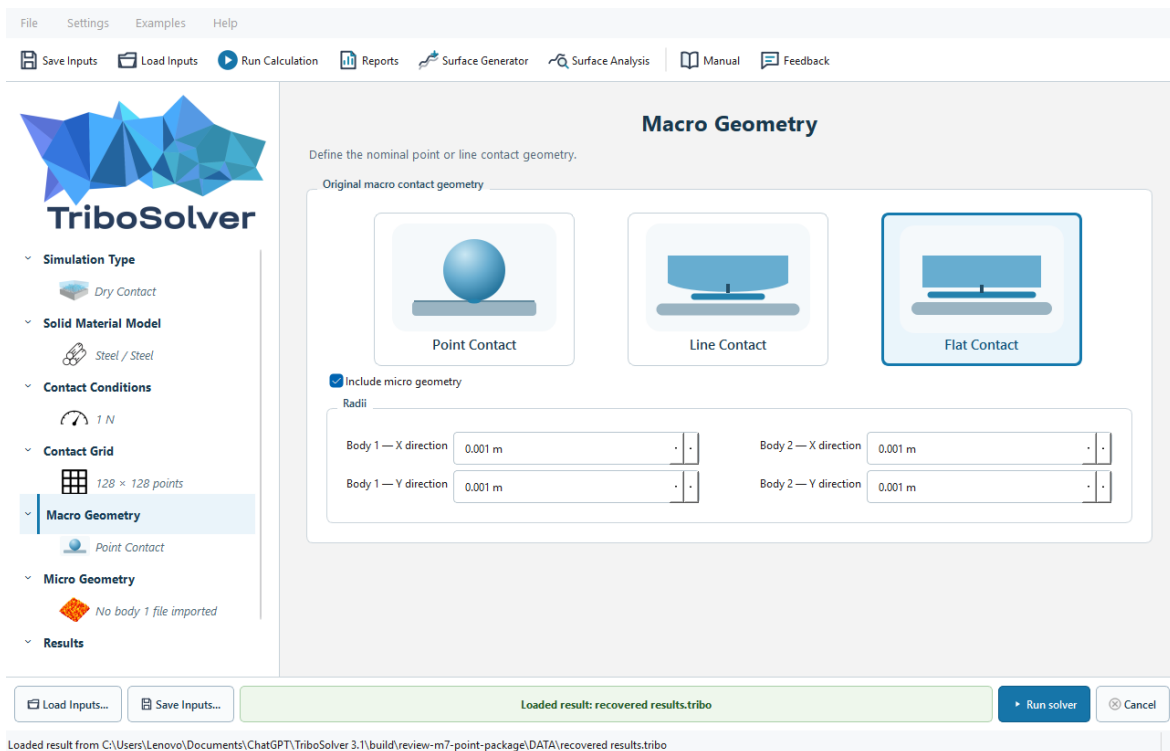


Figure 6-2. Macro Geometry: select the contact shape and enter its applicable dimensions.

6.3 MICRO GEOMETRY

On the Micro Geometry tab, set the roughness resolution, tilt correction and whether to include one or both surfaces. Surface Generator on the toolbar opens a separate window for generating and previewing a surface. Close the window to return to the setup; the generated surface remains available for the calculation. Surface Analysis is also available on the toolbar.

Surface Analysis accepts complete rectangular CSV/TXT XYZ grids; the number of samples need not be a square. Use three columns containing X, Y and height in metres, with uniform spacing along each axis; X and Y spacing may differ. Rows may appear in any order. Heights are mapped to their coordinate pairs, with X along columns and Y along rows. This corrects the transposed orientation of older headerless XYZ imports; previously saved inputs retain their stored surface. Duplicate or missing coordinate pairs and irregular spacing are rejected. At least two samples per axis are needed; very small grids do not establish reliable spectral or correlation estimates. An unheaded 3 by 3 table remains a height matrix. Use the first-line header `x_m,y_m,height_m` to explicitly identify XYZ, or `height_matrix` to identify a three-column height matrix, then enter Step X and Step Y for matrix data. The DAT format retains its existing square-grid convention.

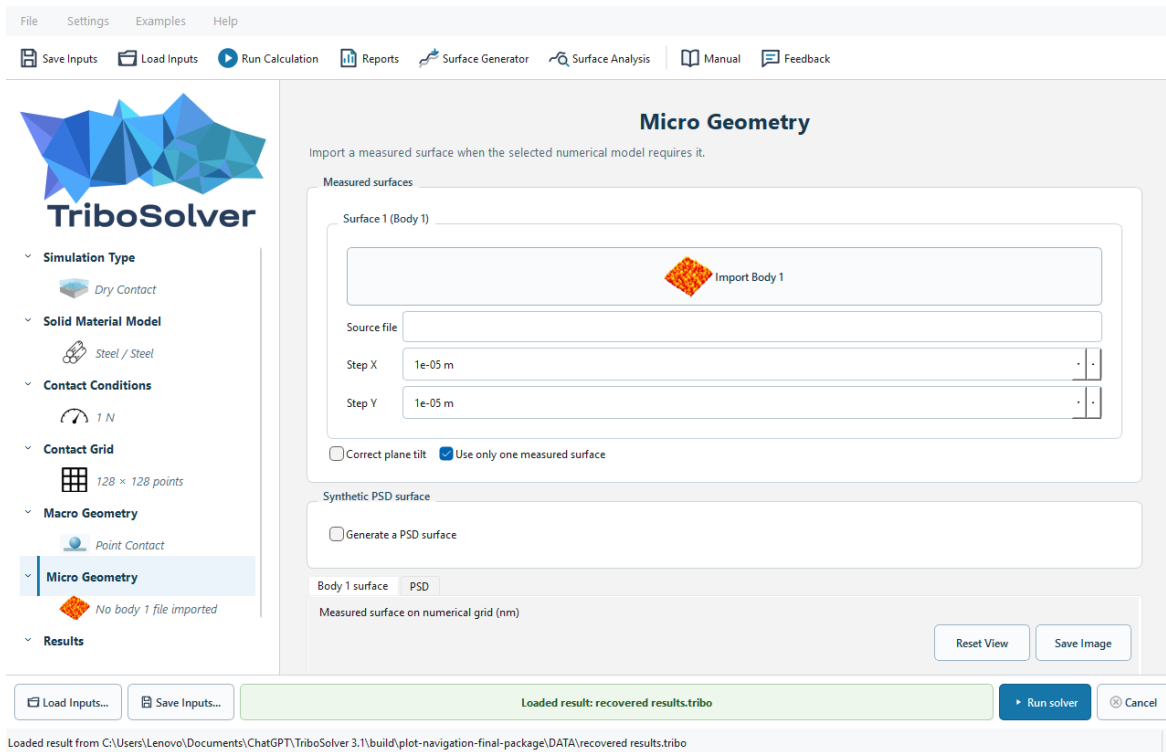


Figure 6-5. Micro Geometry: load surface data or configure a generated surface.

6.4 SOLID MATERIAL MODEL TAB

On the Solid Material Model Tab, material properties of contacting bodies need to be specified. If Include Coatings flag is checked, then the properties and the thickness of the coatings needs to be specified.

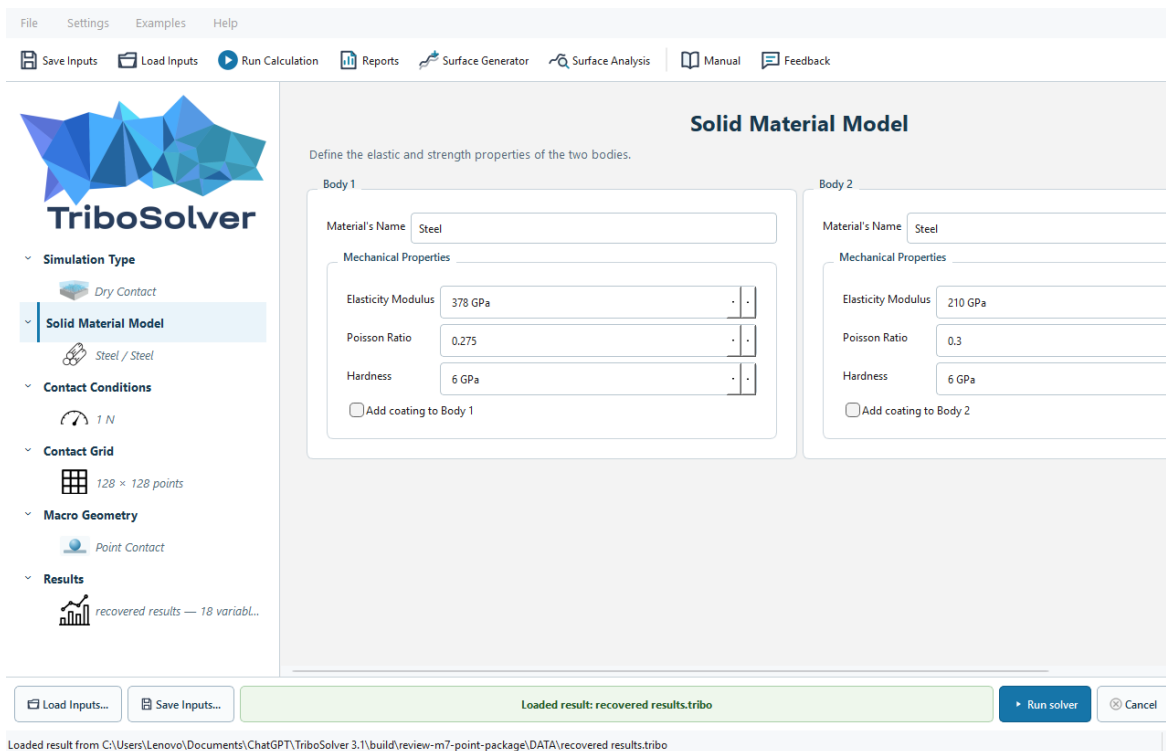


Figure 6-6. Solid Material Model: enter the properties of the contacting bodies.

6.5

CONTACT CONDITIONS TAB

On the Contact Conditions tab the inputs describing the loads, speed, boundary conditions are specified.

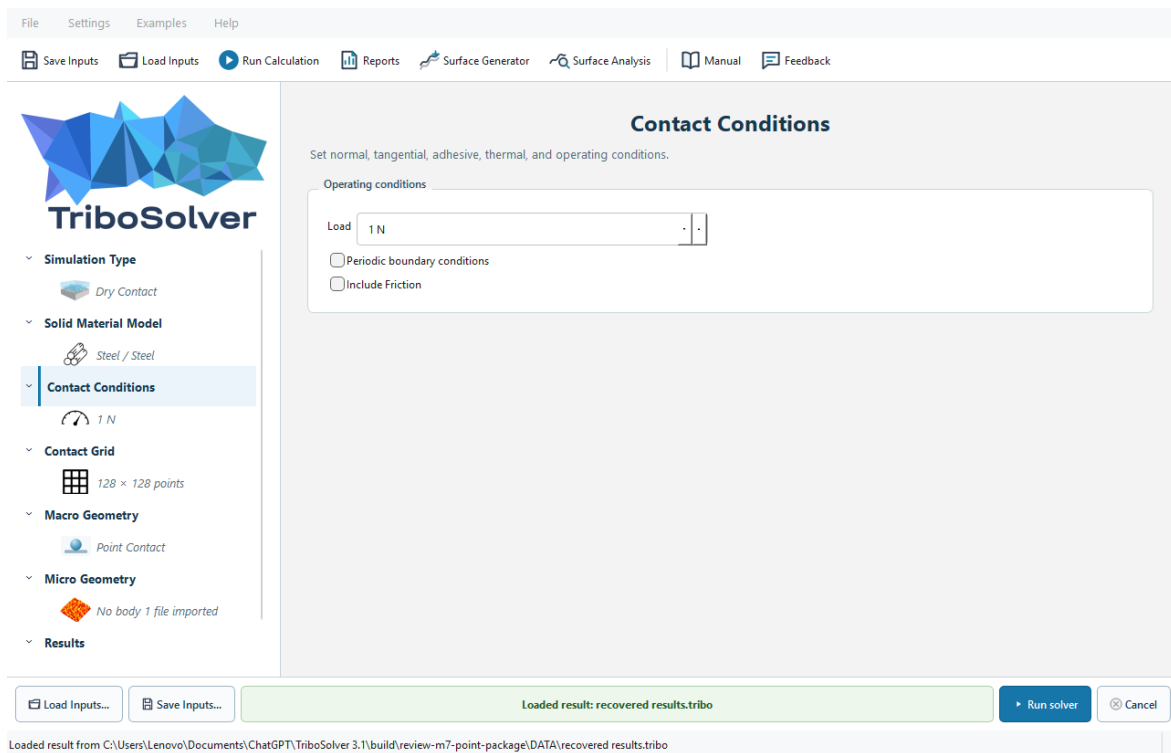


Figure 6-7. Contact Conditions: enter the load and the applicable operating conditions.

6.6

ADHESION AND ELECTROADHESION INPUTS

Choose Adhesion on the Simulation Type tab. On the Adhesion tab, enable surface adhesion, electroadhesion, or both. Surface adhesion uses adhesion energy and interaction range in addition to the common geometry, material, grid and load inputs.

Electroadhesion calculations require applied voltage, the dielectric thickness and relative permittivity for each body, and the relative permittivity of the inter-body space. A warning threshold does not make a case safe; material-specific breakdown and operating limits remain the user's engineering responsibility.

6.7

CONTACT GRID TAB

On the Contact Grid tab a numerical grid for dry contact solver is specified. It is recommended to choose the numerical grid to be the same, as the grid of the surface roughness measurement data, if applicable. If not possible, the surface roughness will be interpolated to the contact grid.

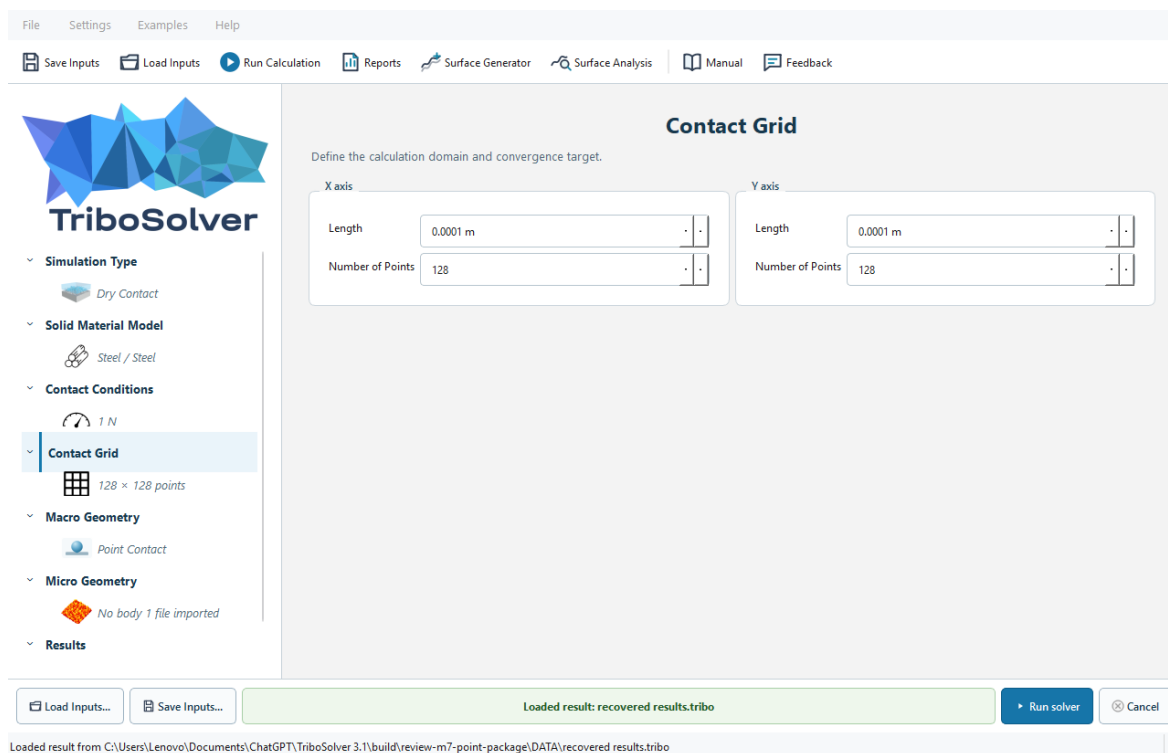


Figure 6-8. Contact Grid: configure the calculation domain and resolution.

6.8 LUBRICANT MODEL TAB

On Lubricant Model, specify the lubricant properties required by the selected lubricated calculation. Review the units displayed beside each input.

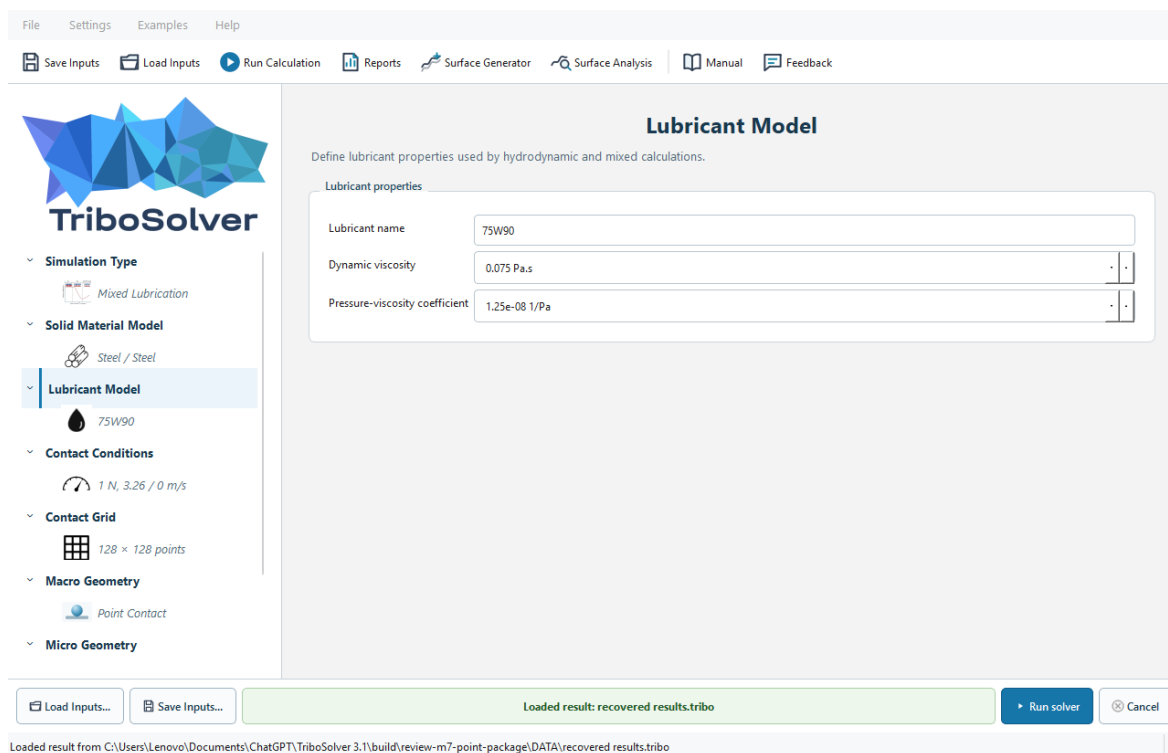


Figure 6-9. Lubricant Model page of TriboSolver.

7.

OUTPUT VARIABLES AND RESULT FILES

7.1

TRIBOSOLVER 3.1 RESULT SETS

Select Manual on the toolbar or Help > Manual to open this full manual offline. The File menu contains Load Inputs, Save Inputs, Load Results and Exit. Surface import is available within Micro Geometry; result export controls are on Results. On smaller windows, Results controls appear above the plot. Scroll vertically to reach the remaining controls and plot. Hover over a long variable name or folder path to read it in full; filename and folder fields also support keyboard selection and copying.

Scroll over a graph to zoom in or out. On line plots and maps, the point under the pointer stays in place until the data boundary is reached. Scroll outside the plot to move the page. Use Reset View to restore the full data range and the original 3D orientation; drag a 3D surface with the left mouse button to rotate it. Changing the plotted data resets the view.

Save Image saves a PNG of the current plot, including its zoom, 3D orientation, selected variable or stress section, title, axes, units and any legend. The saved image excludes application buttons and hover popups. Choose a filename and confirm replacement if the file already exists. Cancel leaves files unchanged; an unsuccessful write displays an error. Numeric CSV exports and Save Results keep their existing data scope: zoom does not crop or change their values. Plot minimum and maximum annotations describe the complete plotted dataset.

A successful calculation is automatically saved in the DATA directory beside TriboSolver.exe when that location is writable. Before running, check Results filename. If the target result or companion CSV already exists, a confirmation identifies the files to replace. Choose Yes to run and replace them after a successful calculation, or Cancel to return to setup and choose another name. Check the save status before closing. On Results, Save Results preserves the complete dataset in a .tribo file. File > Load Results reopens saved results, including supported legacy result files. CSV exports the selected variable, body and section; Export report creates a report for the selected output. Input settings use .tribo.json and are saved separately with File > Save Inputs and re-opened with File > Load Inputs.

Adhesive result sets include total, repulsive and adhesive pressure fields; elastic deformation and gap; repulsive-contact and adhesive-zone masks when applicable; load components; contact and adhesive-zone areas; pressure extrema; energy, residual and iteration histories supported by the selected model. Electroadhesion result sets include signed electric field, signed surface charge density, attractive traction and active-gap mask, together with effective electrical thickness, interface voltage, total attraction force, field and charge extrema, active gap area, breakdown warning, contact area ratio, apparent capacitance, apparent friction coefficient and nonlinear solver history. Average repulsive pressure is the repulsive force divided by repulsive contact area; average

electroadhesive pressure is the signed electroadhesive force divided by the electrically active gap area.

7.2 INSPECTING AND SAVING RESULTS

Choose the output variable on Results. Scalar outputs are shown as values. Field and volume outputs provide the applicable plot and section controls, with units on the axes. For subsurface results, select the body, variable and depth or section; the original contact-pressure result remains available. Contact maps use black for contact and light gray for non-contact. Save Results preserves the complete dataset, while CSV and reports follow the selected output. At smaller window sizes, scroll vertically to reach the result controls. Result groups separate physical outputs from convergence and numerical diagnostics. Calculation messages remain visible above the plots. Numerical convergence alone does not establish physical validity or grid independence. Save/export guidance below the Results buttons identifies the selected output and starting folders; choose the final location in the save dialog. A 1D graph view does not crop the selected variable CSV. PDF includes the result inventory and selected output; DOCX includes the selected output and run setup.

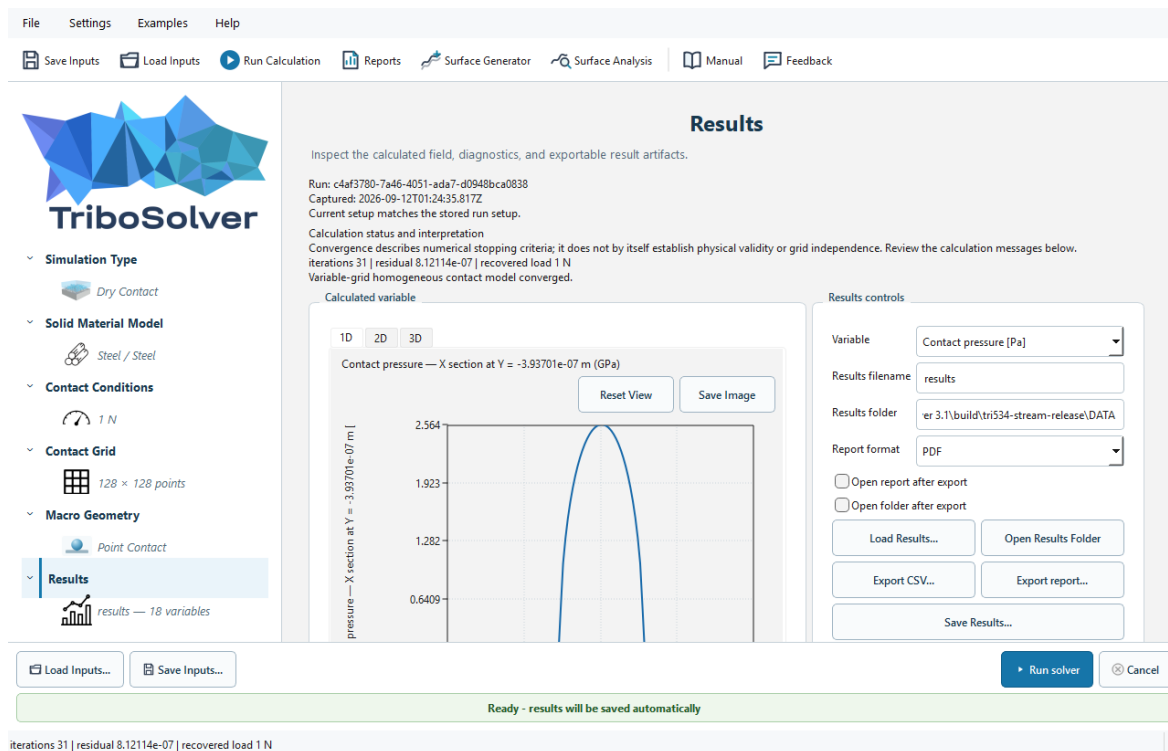


Figure 7-1. Results workspace and output controls.

7.3

SURFACE PREPARATION AND RECOVERED INPUTS

Micro Geometry shows source sample counts, spacing and sampled span separately from the prepared array and requested calculation grid. The sampled span is spacing multiplied by one less than the sample count; calculation spacing is domain length divided by point count. Preparation uses linear interpolation and mirrored extension. Embedded snapshots describe their retained grid without claiming to retain the original measured-file sampling.

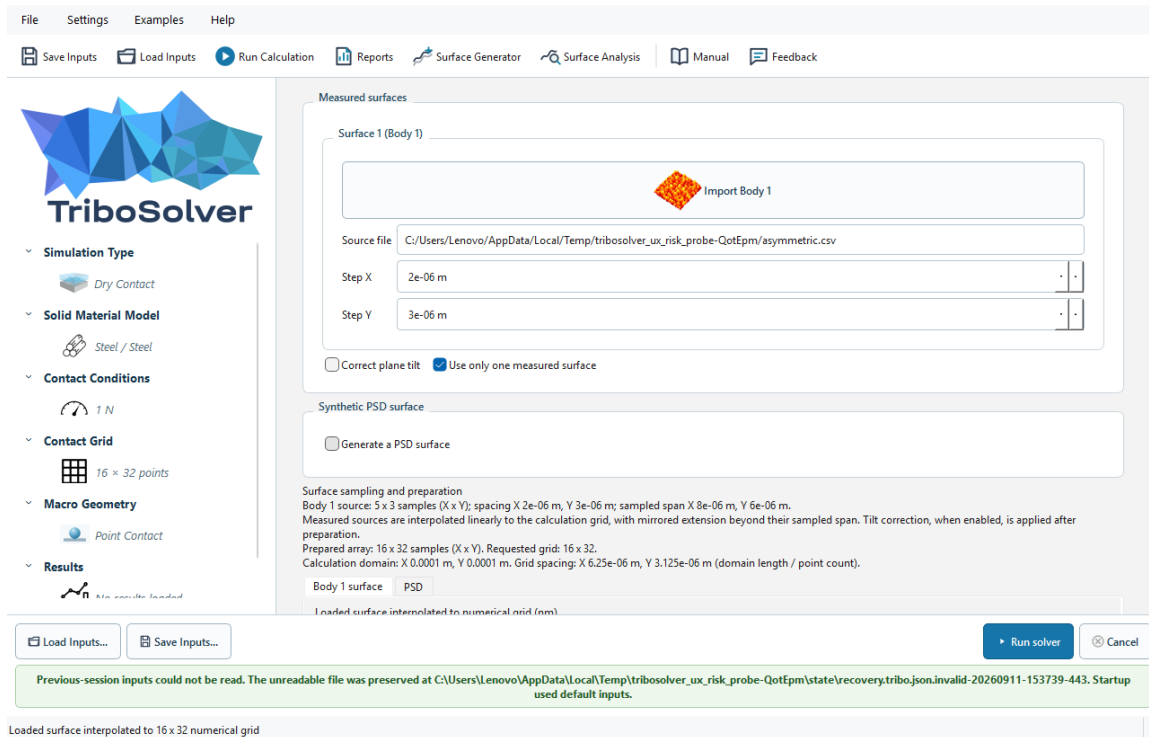


Figure 7-2. Surface source sampling and prepared calculation grid.

Save Inputs reports the saved location; results are saved separately. Previous-session inputs restore automatically. If a snapshot is unreadable, TriboSolver preserves a copy and starts with defaults. If preservation fails, automatic recovery saving pauses to protect the original file. Read the status message and use Save Inputs to a writable folder. Failed input writes preserve the previous file.

7.4

COMPARING SAVED RESULTS

On Results, use Load Results to import one or more saved native result files. Each imported result appears under Results in the left sidebar. Check or clear its box to include or exclude its curve, edit its display label, or select it and use Remove selected dataset. The original saved files remain unchanged.

In Variable, select the required quantity marked imported results. Compatible checked datasets are plotted together in the main Results area. Choose the X or Y direction and a shared physical section position in metres; volume quantities also use the selected depth. Compatible units are converted explicitly. Linear interpolation across section positions and depths retains native along-section coordinates and never extrapolates beyond a dataset. Missing quantities or unavailable positions are explained in the plot notes. The current result retains its individual variable entries for the standard views and numeric/report exports.

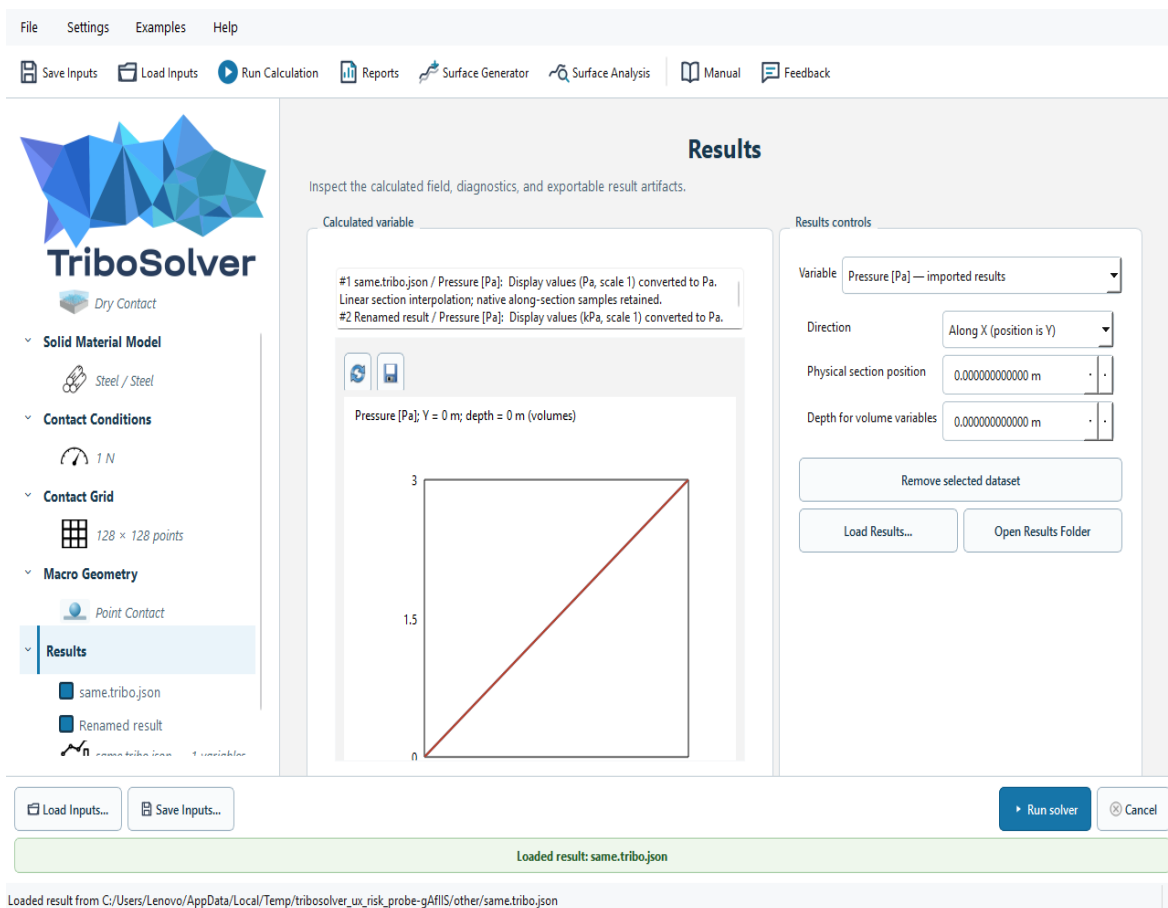


Figure 7-3. Imported results in the sidebar and shared cross-section plot in the Results page.

Scroll within the plot area to reach its full legend. Use the wheel to zoom horizontally. The small Reset View and Save Image icons restore the full range and export a PNG with title, axes, units and full legend. Hover over an icon to read its tooltip. Comparison does not rerun simulations or modify numeric datasets.

8.

TRIBOSOLVER IN LITERATURE

TriboSolver was used for research and several articles were published based on the results obtained by Tribology Simulator (the predecessor of TriboSolver). Here is the list of the relevant articles:

- Feng-Chun Hsia, Chao-Chun Hsu, Liang Peng, Fiona M. Elam, Chen Xiao, Steve Franklin, Daniel Bonn, and Bart Weber, [Contribution of Capillary Adhesion to Friction at Macroscopic Solid-Solid Interfaces](#), Phys. Rev. Applied 17, 034034 – Published 11 March 2022
- Feng-Chun Hsia, Fiona M. Elam, Daniel Bonn, Bart Weber, Steve E. Franklin, Wear particle dynamics drive the difference between repeated and non-repeated reciprocated sliding, <https://doi.org/10.1016/j.triboint.2019.105983>
- B. Weber and T. Suhina and A. M. Brouwer and D. Bonn, Frictional weakening of slip interfaces, <https://www.science.org/doi/abs/10.1126/sciadv.aav7603>
- Popov VL (2018) Is Tribology Approaching Its Golden Age? Grand Challenges in Engineering Education and Tribological Research. Front. Mech. Eng. 4:16. doi: 10.3389/fmech.2018.00016
- Aydar Akchurin, Rob Bosman, Piet M. Lugt & Mark van Drogen, On a Model for the Prediction of the Friction Coefficient in Mixed Lubrication Based on a Load-Sharing Concept with Measured Surface Roughness, 2015, <https://link.springer.com/article/10.1007/s11249-015-0536-z> .

9.

APPENDICES

9.1

APPENDIX 1

Expression for the contact area of i -th asperity in deterministic elastic-plastic model is:

$$A_i(\delta_i) = \begin{cases} A_{ie}, & \delta_i \leq \delta_{ie} \\ A_{iep}, & \delta_{ie} \leq \delta_i \leq \delta_{ip} \\ A_{ip}, & \delta_i \geq \delta_{ip} \end{cases} \quad (24)$$

Where

$$\begin{aligned} A_{ie} &= \pi \delta_i \beta_i \\ A_{ip} &= 2 \pi \delta_i \beta_i \\ A_{iep} &= \pi \delta_i \beta_i \left(1 - 2 \left(\frac{\delta_i - \delta_e}{\delta_p - \delta_e} \right)^3 + 3 \left(\frac{\delta_i - \delta_e}{\delta_p - \delta_e} \right)^2 \right) \end{aligned} \quad (25)$$

According to Greenwood and Williamson, critical indentation of the asperity $\delta_{ie} = \left(0.94 \frac{H}{E} \right)^2 \delta_i$, with

H – hardness of the softer material. When δ_i is larger than δ_{ie} , then the elastic-plastic deformation

regime occurs. Fully plastic regimes starts when $\delta_i > 54 \delta_{ie}$, therefore $\delta_{ip} = 54 \delta_{ie}$. Total area of contact is then $A = \sum_1^N A_i(\delta_i)$. Equation for the load carried by the i -th asperity is:

$$F_i(\delta_i) = \begin{cases} F_{ie}, & \delta_i \leq \delta_{ie} \\ F_{iep}, & \delta_{ie} \leq \delta_i \leq \delta_{ip} \\ F_{ip}, & \delta_i \geq \delta_{ip} \end{cases} \quad (26)$$

And total carried load is $\tilde{F}_C = \sum_1^{Ne} F_{ie} + \sum_1^{Nep} F_{iep} + \sum_1^{Np} F_{ip}$ with

$$\begin{aligned} F_{ie} &= \frac{2}{3} E' \beta_i^{\frac{1}{2}} \delta_i^{3/2} \\ F_{ip} &= H A_{ip} \\ F_{iep} &= \left(H - 0.6 H \frac{\ln \delta_{ip} - \ln \delta_i}{\ln \delta_{ip} - \ln \delta_{ie}} \right) A_{iep} \end{aligned} \quad (27)$$

10. REFERENCES

- Akchurin, A., & Bosman, R. (2017). A Deterministic Stress-Activated Model for Tribo-Film Growth and Wear Simulation. *Tribology Letters*, 65(2). <https://doi.org/10.1007/s11249-017-0842-8>
- Akchurin, A., Bosman, R., & Lugt, P. M. (2016). A Stress-Criterion-Based Model for the Prediction of the Size of Wear Particles in Boundary Lubricated Contacts. *Tribology Letters*, 64(3). <https://doi.org/10.1007/s11249-016-0772-x>
- Akchurin, Aydar, & Bosman, R. (2017). A Deterministic Stress-Activated Model for Tribo-Film Growth and Wear Simulation. *Tribology Letters*, 65(2), 1–9. <https://doi.org/10.1007/s11249-017-0842-8>
- Bares, J. A., Konicek, A. R., Mangolini, F., Gosvami, N. N., Carpick, R. W., & Yablon, D. G. (2015). Mechanisms of antiwear tribofilm growth revealed in situ by single-asperity sliding contacts. *Science*, 348(6230), 102–106.
- Bhushan, B. (2013). *Principles and Applications of Tribology, 2nd edition*. New York: A Wiley-Interscience Publication.

- Bugnicourt, R., Sainsot, P., Dureisseix, D., Gauthier, C., & Lubrecht, A. A. (2018). FFT-Based Methods for Solving a Rough Adhesive Contact: Description and Convergence Study. *Tribology Letters*, 66, 29. <https://doi.org/10.1007/s11249-017-0980-z>
- Bos, J., & Moes, H. (1995). *Frictional Heating of Tribological Contacts* (University of Twente; Vol. 117). <https://doi.org/10.1115/1.2830596>
- Fujita, H., Glovnea, R. P., & Spikes, H. A. (2005). Study of zinc dialkydithiophosphate antiwear film formation and removal processes, part I: Experimental. *Tribology Transactions*, 48(4), 558–566. <https://doi.org/10.1080/05698190500385211>
- Hsia, F.-C., Hsu, C.-C., Peng, L., Elam, F. M., Xiao, C., Franklin, S., Bonn, D., & Weber, B. (2022). Contribution of Capillary Adhesion to Friction at Macroscopic Solid-Solid Interfaces. *Physical Review Applied*, 17, 034034. <https://doi.org/10.1103/PhysRevApplied.17.034034>
- Hughes, T. G., Elcoate, C. D., & Evans, H. P. (1999). Coupled Solution of the Elastohydrodynamic Line Contact Problem Using a Differential Deflection Method. *Proceedings of the Institution of Mechanical Engineers, Part C*, 214(C), 585–598.
- Jacobs, T., & Martini, A. (2017). Review : Measuring and Understanding Contact Area at the Nanoscale. *Applied Mechanics Reviews*, (c). <https://doi.org/10.1115/1.4038130>
- Jacq, C., Nélías, D., Lormand, G., & Girodin, D. (2002). Development of a Three-Dimensional Semi-Analytical Elastic-Plastic Contact Code. *Journal of Tribology*, 124(October), 653. <https://doi.org/10.1115/1.1467920>
- Johnson, K. L. (1984). *Contact mechanics*. [https://doi.org/10.1016/0301-679X\(86\)90085-X](https://doi.org/10.1016/0301-679X(86)90085-X)
- Johnson, K. L., Greenwood, J. A., & Poon, S. Y. (1972). A simple theory of asperity contact in elastohydro-dynamic lubrication. *Wear*, 19(1), 91–108. [https://doi.org/10.1016/0043-1648\(72\)90445-0](https://doi.org/10.1016/0043-1648(72)90445-0)

- King, R. B., & Sullivan, T. C. O. (1987). *SLIDING CONTACT STRESSES IN A TWO-DIMENSIONAL LAYERED ELASTIC HALF-SPACE of the stress state in layered media under combined sliding and normal contact is important for assessing mechanical reliability . Applications include protective coatings , layered.*
- Lechthaler, B., Ochs, G., Mücklich, F., & Dienwiebel, M. (2019). Evolution of the True Contact Area of Laser Textured Tungsten Under Dry Sliding Conditions. *Frontiers in Mechanical Engineering*, 5(February), 1–6. <https://doi.org/10.3389/fmech.2019.00003>
- Liu, S., & Wang, Q. (2002). Studying Contact Stress Fields Caused by Surface Traction With a Discrete Convolution and Fast Fourier Transform Algorithm. *Journal of Tribology*, 124(1), 36. <https://doi.org/10.1115/1.1401017>
- Love, B. A. E. H. (1929). *The Stress produced in a Semi-infinite Solid by Pressure on Part of the Boundary.* 228(1929), 377–420.
- Ludema, K. C. (1996). *Friction, wear and Lubrication A textbook in Tribology.* Ann Arbor: CRC Press.
- Maugis, D. (1992). Adhesion of Spheres: The JKR-DMT Transition Using a Dugdale Model. *Journal of Colloid and Interface Science*, 150(1), 243–269. [https://doi.org/10.1016/0021-9797\(92\)90285-T](https://doi.org/10.1016/0021-9797(92)90285-T)
- Meng, H. C., & Ludema, K. C. (1995). Wear models and predictive. *Wear*, 181–183, 443–457. [https://doi.org/doi:10.1016/0043-1648\(95\)90158-2](https://doi.org/doi:10.1016/0043-1648(95)90158-2)
- Müser, M. H., Dapp, W. B., Bugnicourt, R., Sainsot, P., Lesaffre, N., Lubrecht, T. A., ... Greenwood, J. A. (2017). Meeting the Contact-Mechanics Challenge. *Tribology Letters*, 65(4). <https://doi.org/10.1007/s11249-017-0900-2>
- Peng, L., Kooij, S., Ciftci, H. T., Bonn, D., & Weber, B. (2026). Strong Electroadhesion at Low Voltage Enabled by Nanoscale Roughness. *Physical Review Research*, 8, 033143. <https://doi.org/10.1103/l1v9-fpps>

- Polonsky, I. A., & Keer, L. M. (1999). A numerical method for solving rough contact problems based on the multi-level multi-summation and conjugate gradient techniques. *Wear*, 231(2), 206–219. [https://doi.org/10.1016/S0043-1648\(99\)00113-1](https://doi.org/10.1016/S0043-1648(99)00113-1)
- Rey, V., Anciaux, G., & Molinari, J. F. (2017). Normal adhesive contact on rough surfaces: efficient algorithm for FFT-based BEM resolution. *Computational Mechanics*, 60(1), 69–81. <https://doi.org/10.1007/s00466-017-1392-5>
- Timoshenko, S. P., & Goodier, J. N. (1970). *Theory of Elasticity* (3rd-rd ed.). New York: McGraw-Hill.
- Weber, B., Suhina, T., Brouwer, A. M., & Bonn, D. (2019). Frictional weakening of slip interfaces. *Science Advances*, 5(4), 1–8. <https://doi.org/10.1126/sciadv.aav7603>
- Yastrebov, V. A., Anciaux, G., & Molinari, J. F. (2014). The contact of elastic regular wavy surfaces revisited. *Tribology Letters*, 56(1), 171–183. <https://doi.org/10.1007/s11249-014-0395-z>
- Zhang, J., & Spikes, H. (2016). On the Mechanism of ZDDP Antiwear Film Formation. *Tribology Letters*, 63(2), 1–15. <https://doi.org/10.1007/s11249-016-0706-7>

End of document